Full Pressure Coupling for Geo-mechanical Multi-phase Flow Simulations in Porous Media

Florian Doster[1], Nathalia Passos[2] and Jan M. Nordbotten[3]

1: Institute of Petroleum Engineering, Heriot Watt University, UK; 2:Dep. Den Engenharia, Pontifica Universidade Catolica do Rio de Janeiro, BR 3: Dep. of Mathematics, University of Bergen, NO

1 ABSTRACT

Unconventional subsurface exploration pushes operational envelope of geological media.

➔ Rising demand for coupled flow and geo-mechanics simulation

Coupled simulations remain a challenge:

- Different physics and different mathematical structure of equations favour different discretisation techniques.
- Common software packages designed for either flow or geo-mechanical simulations

Coupling categories (e.g. [1]):

1. Decoupled
2. Iteratively coupled
3. Fully coupled

↓ Increase in stability and accuracy but also in implementation effort

Despite disadvantages, decoupled and iteratively coupled schemes are preferred in practice because they allow for using established software packages (see e.g. [2]).

Full pressure coupling

Here a new coupling coupling scheme for iterative coupling is introduced.

Strategy:

1. Solve geo-mechanics fully coupled to a single-phase flow problem using global pressure
2. Couple the resulting deformation to a multi-phase multi-component flow solver iteratively.

Features:

1. No new software development required because most geo-mechanic simulators include fully coupled single-phase flow solvers
2. Overhead in pressure solve: Splitting scheme requires solving the pressure field twice. BUT: Computational costs of solvers for deformation and non-linear transport of flow dominate. Overhead per iteration step small and justified if the number of iterations is small.

ImPDEM: Implicit Pressure and Deformation Explicit Masses

The full pressure coupling scheme is illustrated through a finite volume Implicit Pressure and Deformation Explicit Masses (ImPDEM) scheme.

ImPDEM is based on a finite volume discretisation for geo-mechanics and single-phase flow [3] and a robust finite volume Implicit Pressure Explicit Masses (ImPEM) method for multi-phase multi-component flow [4].

For ImPDEM time-stepping the coupling is exact and the iterative scheme converges in one iteration, even for nonlinear flow and nonlinear geo-mechanics.

The finite volume discretization of stress and strain enables a seamless integration of geo-mechanical phenomena into ImPEM. ➔ No redundant solving for pressure.

Advanced transport and thermodynamic solvers can be applied without additional modifications.

2 MODEL EQUATIONS AND FV-IMPDEM DISCRETISATION

Momentum conservation of porous medium

$$\int_\omega \rho_s \frac{\partial^2}{\partial t^2}\mathbf{D}dV = \int_{\partial\omega} \mathbf{T_n} dS - \int_\omega \rho_s \mathbf{g} dV$$

Mass conservation of fluids

$$\frac{d}{dt}\int_\omega m_f\, dV + \int_{\partial\omega} \rho_f\, w_{f_\mathbf{n}}\, dS = \int_\omega q_f\, dV$$

Biot's law

$$\sigma = \mathbb{C} : \frac{1}{2}\left(\nabla\mathbf{D} + (\nabla\mathbf{D})^T\right) - \alpha\, p\mathbf{1}$$

Darcy's law

$$\mathbf{w_f} = -\mathbf{K}\lambda_f\left(\nabla p_f - \rho_f\, \mathbf{g}\right)$$

Capillary pressure

$$p_{fg}^{\mathrm{cap}} = p_f - p_g$$

Volume conservation

$$\sum_f s_f = 1$$

Surface stress

$$\mathbf{T_n} = \sigma \cdot \mathbf{n}$$

Fluid mass density

$$m_f = s_f \rho_f(p)\, \phi(\nabla\cdot\mathbf{D})$$

Surface flux

$$w_{f_\mathbf{n}} = \mathbf{w_f}\cdot\mathbf{n}$$

Fluid pressure

$$p = p(p_f)$$

Discrete elastic equations

$$\sum_j \frac{|\partial\omega_{ij}|}{|\omega_i|}\tilde{\mathbf{T}}_{ij} + \rho_s\, \mathbf{g} = \alpha\frac{1}{|\omega_i|}\int_{\partial\omega_i} p\mathbf{n}\, dS$$

Discrete pressure equations

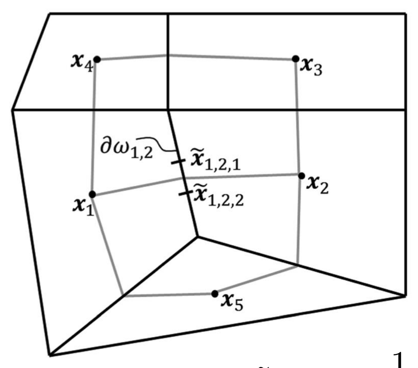
$$\phi_i \sum_f c_f \frac{dp_i}{dt} + \sum_j \frac{|\partial\omega_{ij}|}{|\omega_i|}\sum_f f_{f\,ij} - \sum_f q_{f\,i} = -\alpha\frac{d}{dt}\left(\frac{1}{|\omega_i|}\int_{\partial\omega_i}\mathbf{D}\cdot\mathbf{n}\, dS\right) + \phi_i\beta V_{\mathrm{res}\,i}$$

Discrete transport equations

$$\frac{dm_{f\,i}}{dt} + \sum_j \frac{|\partial\omega_{ij}|}{|\omega_i|}\rho_{f\,i} f_{f\,ij} - \rho_{f\,i}\, q_{f\,i} = 0$$

Relaxed volume approach

$$\frac{\partial V_{\mathrm{res}}}{\partial t} = -\beta V_{\mathrm{res}} \qquad V_{\mathrm{res}} = \sum_f s_f - 1$$



Discrete fluxes (MPFA)

$$f_{f\,ij} = \frac{1}{|\partial\omega_{ij}|}\int_{\partial\omega_{ij}} w_{f\,\mathbf{n}}\, dS$$

Discrete stresses (MPSA)

$$\tilde{\mathbf{T}}_{ij} = \frac{1}{|\partial\omega_{ij}|}\int_{\partial\omega_{ij}} \mathbb{C} : \frac{1}{2}\left(\nabla\mathbf{D} + (\nabla\mathbf{D})^T\right)\, dS$$

Timestep constraints

Geo-mechanics: Minimum timestep

Explicit transport: Maximum timestep

But: Decoupling of timestep possible through relaxed volume and fractional flow

Schematic geo-mechanic/pressure linear system of equations

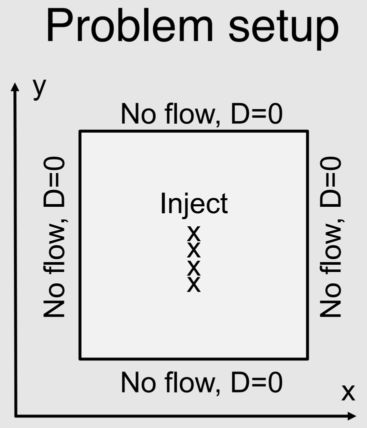
$$\begin{pmatrix} \mathbf{M^D} & -\alpha\mathbf{M}^{\mathrm{grad}} \\ \alpha\mathbf{M}^{\mathrm{div}} & \mathbf{C}^p + \mathbf{M}^p \end{pmatrix}\begin{pmatrix} \mathbf{D}_i^n \\ p_i^n \end{pmatrix} = \begin{pmatrix} \mathbf{G}_{si} \\ Q_i + G_{f\,i} + V_{\mathrm{res}\,i} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \alpha\mathbf{M}^{\mathrm{div}} & \mathbf{C}^p + \mathbf{M}^p \end{pmatrix}\begin{pmatrix} \mathbf{D}_i^{n-1} \\ p_i^{n-1} \end{pmatrix}$$

Schematic transport: $m_{f\,i}^{\,n} = m_{f\,i}^{\,n-1} + \rho_{f\,i}^{\,n}\sum_j \frac{|\partial\omega_{ij}|}{|\omega_i|} f_{f\,ij}^{\,n,n-1} - \sum_f \rho_{f\,i}^{\,n} q_{f\,i}$

3 ILLUSTRATIVE EXAMPLE: CO2 INJECTION INTO CONFINED RESERVOIR

We show the injection of CO_2 into a brine saturated reservoir with no flow boundary conditions on all sides. The displacement is fixed. The injected CO_2 has a compressibility ratio 100 and viscosity ratio of 1/10 compared to the resident brine.

Problem setup



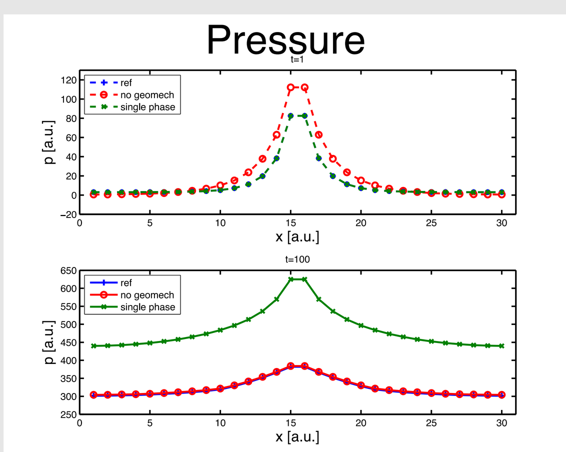
The example illustrates the capability of the method to investigate coupled geo-mechanical multi-phase flow phenomena.

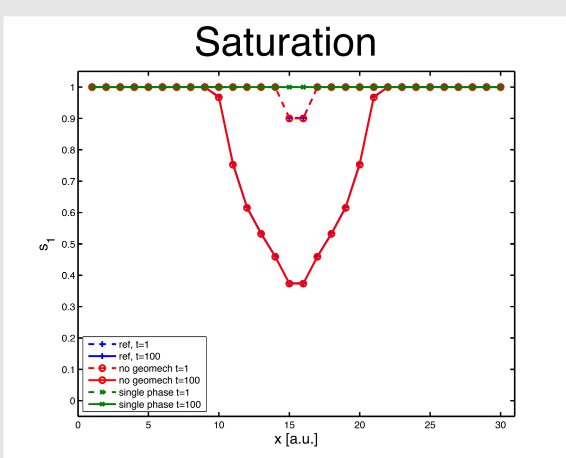
The injection increases the pressure in the reservoir. This induces a deformation which propagates immediately through the domain (geomechanical equilibrium). The deformation couples back onto the pressure: See early pressure profiles.

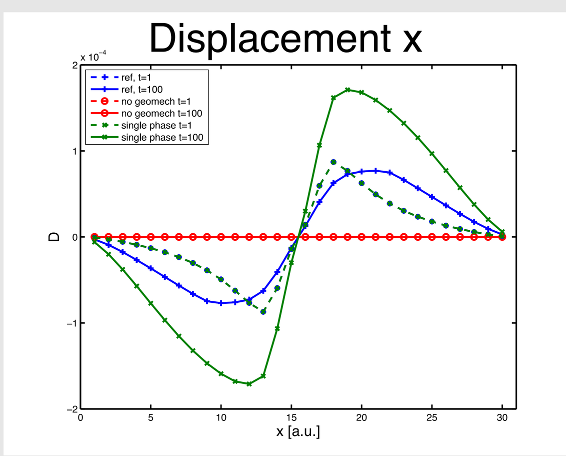
Due to the confinement the pressure in the whole reservoir increases during injection and so does the displacement.

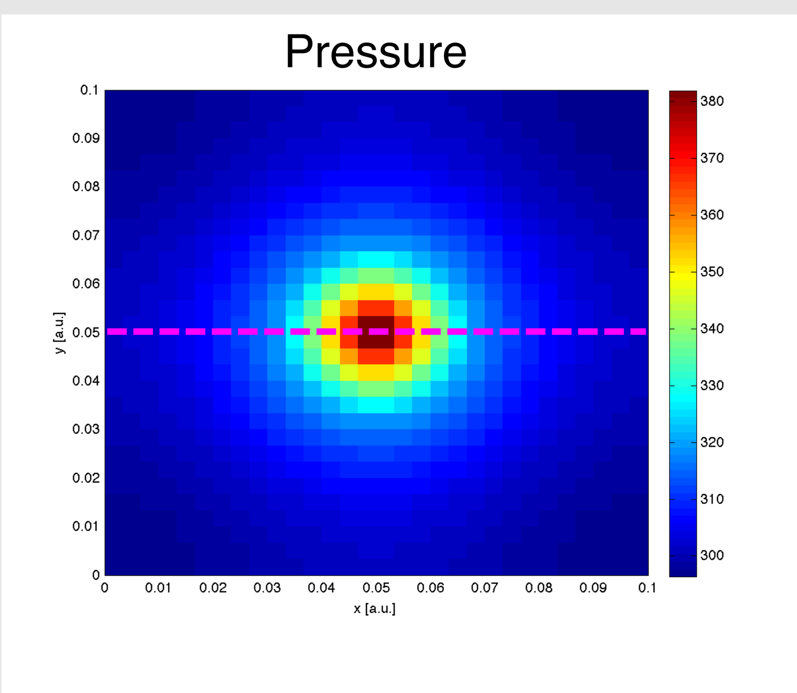
A single phase study overestimates the pressure build up (and therefore also the displacement) because it does not account for the higher compressibility of the injected CO_2.

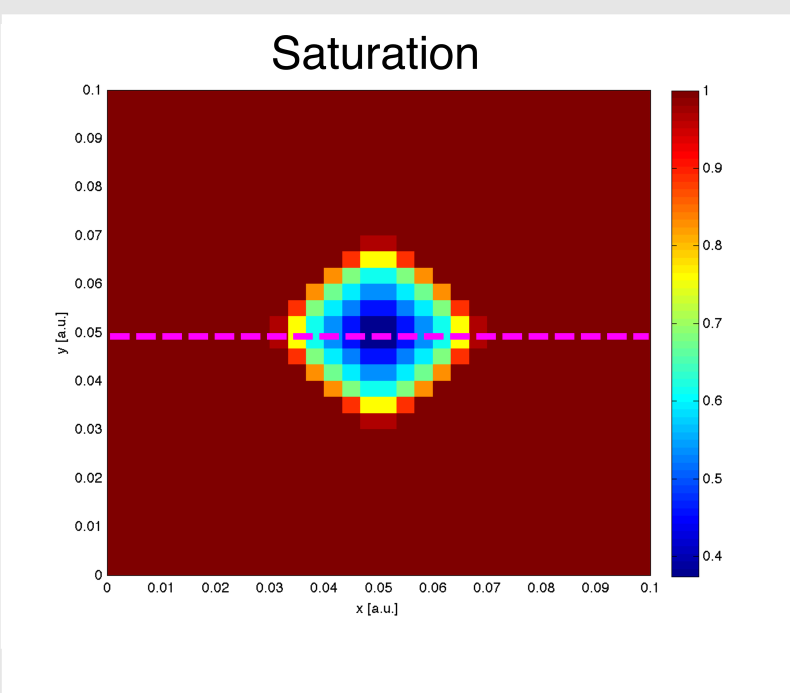
The saturation profiles do not differ because of the no flow boundary conditions.

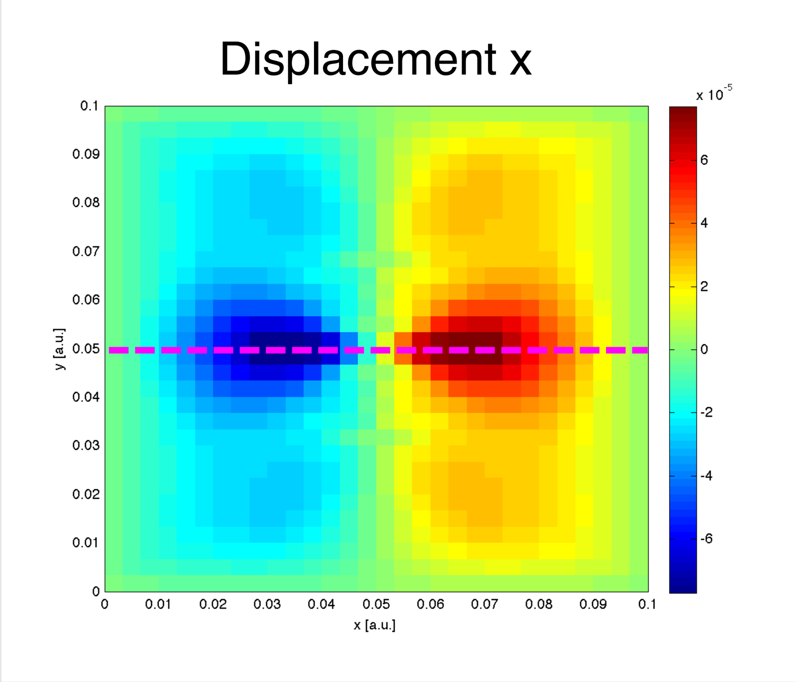


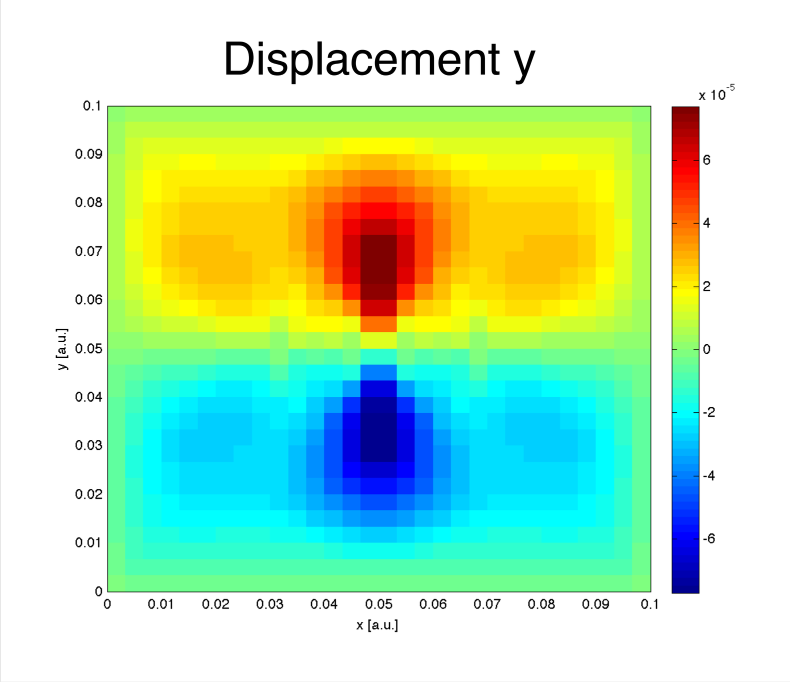












[1] Settari & Walters, SPE 74142 (2001), [2] Pettersen, Int. J. of Num. Analysis & Modeling, 9(3), 628-643 (2012) [3] Nordbotten, WRR, 50(5), 4379-4394 (2014)
[4] Doster, Keilegavlen & Nordbotten, in Al-Khoury & Bundschuh: Computational Models for CO2 Geo-sequestration & Compressed Air Energy Storage, CRC Press (2014)

Florian.Doster@pet.hw.ac.uk